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Supergravity and large-N noncommutative field theories

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Abstract: We consider systems of Dp-branes in the presence of a nonzero B field. We study the corresponding supergravity solutions in the limit where the branes worldvolume theories decouple from gravity. These provide dual descriptions of large N noncommutative field theories. We analyse the phase structure of the theories and the validity of the different description. We provide evidence that in the presence of a nonzero B field the worldvolume theory of D6 branes may decouple from gravity. We analyse the systems of M5 branes and NS5 branes in the presence of a nonzero C field and nonzero RR fields, respectively. Finally, we study the Wilson loops (surfaces) using the dual descriptions.

Keywords: p-branes, Brane Dynamics in Gauge Theories.

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1. Introduction

The AdS/CFT correspondence (see [1] for a review) relates field theories without gravity to supergravity (string) theories on certain curved backgrounds. The correspondence naturally arises when considering Dp-branes in a limit where the world-volume field theory decouples from the bulk gravity [2]. As discussed in [3] and further studied in [4], when turning on a B field on the D-brane worldvolume the low energy effective worldvolume theory is deformed to a noncommutative super-Yang-Mills (NCSYM) theory. With N coinciding Dp-branes in the presence of a nonzero B field the worldvolume theory is deformed to a U(N) NCSYM [5].

Turning on a B field on the D-brane worldvolume can be viewed via the AdS/CFT correspondence as a perturbation of the worldvolume field theory by a higher dimension operator. The noncommutative effects are relevant in the UV and are negligible in the IR. In fact, there is a map from the commutative field theory variables to the noncommutative ones [5]. As in the cases with $B = 0$, there exists a limit where the bulk gravity decouples from the worldvolume noncommutative field theory [6, 5], and a correspondence between string theory on curved backgrounds

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with B field and noncommutative field theories is expected. The aim of this paper is to study this correspondence using Dp-branes, M5 branes and NS5 branes. Related works along this directions are [7, 8, 9]. Other recent studies of noncommutative field theories and string theory are [10].

The paper is organized as follows. In section 2 we will review the effect of a B field on the worldvolume theory of branes. We will discuss the Dp-branes supergravity solutions in the presence of a B field, the decoupling limit and various aspects of the correspondence with the noncommutative worldvolume field theories. We will analyse the phase structure of the Dp-branes and plot their phase diagrams. We will see that the structure can vary depending on the rank of the B field, i.e. depending on the number of noncommutative coordinates. We will argue that, unlike the $B = 0$ case, in the presence of a nonzero B field there is a limit where the worldvolume theory of Dp-branes with $p > 5$ decouples from gravity. In particular, for D6 branes we will see that with two noncommutative coordinates we have for finite N a UV description in terms of eleven dimensional supergravity on a curved space. For four or six noncommutative coordinates we find for finite N a UV description in terms of ten dimensional supergravity on a curved space.

In section 3 we will discuss M5 branes in the presence of a nonzero C field and NS5 branes in the presence of nonzero RR fields. In the case of M5 branes wrapping a circle we will see the same decoupling limit discussed in [5] arising from supergravity. However, in the UV the good description of this system is in terms of D4 branes background, and we do not find a six dimensional field theory description. Considering M5 branes with six flat nocompact worldvolume coordinates we curiously find another decoupling limit. At low energies the supergravity background is of the form $AdS_7 \times S^4$ with a self-dual C field which is the dual description of the $(0, 2)$ theory. As we increase the energy the background is deformed and the C field is no longer self-dual. In section 4 we will use the dual description in order to compute Wilson loops and Wilson surfaces for the different brane theories. We will show that, in some cases, in the presence of the nonzero B (C) field there is way to fix the string (membrane) end point (string) by considering a moving coordinates frame in the computation. Section 5 is devoted to a discussion.

2. Dp-branes in constant B field

2.1 B field background

Consider string theory in flat space in the background of constant NS B field and Dp-branes. In this set up, the end points of the open strings attached to the branes, x_i, are noncommuting [11]:

$$[x_i, x_{i+1}] = il_s^2 \left. \frac{B_{i,i+1}}{1 + B_{i,i+1}^2} \right|_{\text{on the brane}} . \tag{2.1}$$

We will study this system in the limit $B_{i,i+1} \to \infty$ and $l_s \to 0$ such that $b_i \equiv l_s^2 B_{i,i+1}$ is fixed. Rescaling the coordinates $x_i \to \frac{b_i}{l_s^2} x_i$ and keeping the new coordinates fixed in the limit we get $[x_i, x_{i+1}] = ib_i$.

In the presence of the B field, the massless states excitations of the open strings attached to the Dp-branes give rise to a noncommutative worldvolume field theories, with b_i being the deformation parameters. The mode expansions of the open strings coordinates and momenta are:

$$X^i(\sigma,\tau) = x^i + p^i\tau + B^i_j p^j \sigma + \text{oscil.}\,,$$
$$l_s^2\, P^i(\sigma,\tau) = (\mathbf{1} - B^2)^i_j (p^j + \text{oscil.})\,, \tag{2.2}$$

where σ, τ parametrize the string world-sheet [11]. In the above limit the oscillator modes decouple,

$$\bar{X}_i(\sigma) \equiv \frac{b_i}{l_s^2} X_i(\sigma) = \bar{x}_i + b_i \bar{P}_{i+1}\sigma\,, \tag{2.3}$$

where $\bar{P}_i \equiv \frac{l_s^2}{b_i} P_i$ is rescaled in order to preserve the canonical commutator relations.

As we see in (2.3), there is a finite part added to the string end point, which is proportional to the momentum. Physically it means that the open strings attached to a mixed brane are "dipoles" of the worldvolume U(N) gauge theory [12, 13, 14] and this, in part, is a reflection of the non-locality in these theories. The moment of these dipoles are proportional to $b_i P_{i+1}$.

2.2 The string (supergravity) description

In the following we will discuss the dual formulation of noncommutative gauge theories as string (supergravity) theory on curved backgrounds with a non-zero B field. Consider now the supergravity description of Dp-branes in the presence of a non-zero B field. Such solutions were written in [15, 7, 8]. It is straightforward to write the most general solutions. Since we can gauge away the non-zero components of the B field that are normal to the worldvolume of the branes, the relevant cases are those with non-zero components of the B field parallel to the branes. We denote by $2m$, $m = 1, \ldots, [\frac{p+1}{2}]$, the rank of the B field. The space-time coordinates are $x_1, \ldots, x_d$ and we denote by x_{p+1} the time direction.[1] The supergravity background takes the form[2]

$$ds^2 = f_p^{-1/2}\left[\sum_{i\ \text{odd}}^{2m-1} h_i(dx_i^2 + dx_{i+1}^2) + \cdots + dx_{p+1}^2\right] + l_s^4 f_p^{1/2}(du^2 + u^2 d\Omega_{8-p}^2)\,,$$
$$f_p = 1 + \frac{R^{7-p}}{l_s^4 u^{7-p}}\,, \qquad R^{7-p} = c_p g_{YM}^2 N \left(\prod_{i\ \text{odd}}^{2m-1} \cos\theta_i\right)^{-1}\,,$$

[1] For odd p and when $m = [\frac{p+1}{2}]$ we will consider the euclidean signature. As noted in [8], the decoupling limit of the euclidean and lorentzian cases are not the same.

[2] In the following we will not write the RR fields.

$$
\begin{aligned}
h_i^{-1} &= \sin^2\theta_i f_p^{-1} + \cos^2\theta_i\,,\\
B_{i,i+1} &= \frac{\sin\theta_i}{\cos\theta_i} f_p^{-1} h_i\,,\\
e^{2\phi} &= g^2 f_p^{(3-p)/2} \prod_{i\ \mathrm{odd}}^{2m-1} h_i\,,
\end{aligned}
\tag{2.4}
$$

where $c_p = 2^{7-2p}\pi^{\frac{9-3p}{2}}\Gamma(\frac{7-p}{2})$. The energy coordinate u is related to the radial coordinate r by $u = r/l_s^2$ and $g_{YM}^2 = (2\pi)^{p-2} g_s l_s^{p-3}$.

As discussed above, in order to obtain a noncommutative field theory we need to take a limit of infinite B field as $l_s \to 0$. In this limit we keep fixed the parameters $u, \bar{g}_s, b_i, \bar{x}_{i,i+1}$ defined by[3]

$$
\begin{aligned}
u &= \frac{r}{l_s^2}\,, & \bar{g}_s &= g_s l_s^{p-3-2m}\,,\\
b_i &= l_s^2 \tan\theta_i\,, & \bar{x}_{i,i+1} &= \frac{b_i}{l_s^2} x_{i,i+1}\,,
\end{aligned}
\tag{2.5}
$$

where by $x_{i,i+1}$ we mean x_i, x_{i+1}.

In the limit (2.5), the supergravity solution (2.4) reads

$$
\begin{aligned}
l_s^{-2} ds^2 &= \left(\frac{u}{R}\right)^{\frac{7-p}{2}} \left(\sum_{i\ \mathrm{odd}}^{2m-1} h_i (dx_i^2 + dx_{i+1}^2) + \cdots + dx_{p+1}^2\right) + \left(\frac{R}{u}\right)^{\frac{7-p}{2}} (du^2 + u^2 d\Omega_{8-p}^2)\,,\\
R^{(7-p)} &= c_p \bar{g}_{YM}^2 N \prod_{i\ \mathrm{odd}}^{2m-1} b_i\,, \qquad a_i^{7-p} = \frac{b_i^2}{R^{(7-p)}}\,,\\
B_{i,i+1} &= \frac{l_s^2}{b_i} \frac{a_i^{7-p} u^{7-p}}{1 + a_i^{7-p} u^{7-p}}\,, \qquad h_i = \frac{1}{1 + a_i^{7-p} u^{7-p}}\,,\\
e^{2\phi} &= \bar{g}_s^2 \left(\frac{R}{u}\right)^{(7-p)(3-p)/2} \prod_{i\ \mathrm{odd}}^{2m-1} \frac{b_i^2}{1 + a_i^{7-p} u^{7-p}}\,,
\end{aligned}
\tag{2.6}
$$

where

$$
\bar{g}_{YM}^2 = (2\pi)^{(p-2)} \bar{g}_s \sim g_s l_s^{p-3-2m} \tag{2.7}
$$

is the gauge coupling of the noncommutative gauge theory.

The curvature of metric (2.6) in string units

$$
l_s^2 \mathcal{R} \sim \frac{1}{g_{eff}}\,, \tag{2.8}
$$

where g_{eff} is a dimensionless effective gauge coupling of the noncommutative field theory given by

$$
g_{eff}^2 \sim \bar{g}_{YM}^2 N \prod_{i\ \mathrm{odd}}^{2m-1} b_i u^{p-3}\,. \tag{2.9}
$$

[3] For simplicity we will denote in the rest of the paper the rescaled coordinate $\bar{x}_i$ by x_i.

When $g_{eff} \ll 1$ the perturbative field theory description is valid, while when $g_{eff} \gg 1$ the supergravity description is valid. The $l_s^2 \mathcal{R}$ expansion corresponds to the strong coupling expansion in $1/g_{eff}$ of the noncommutative gauge theory. We note that the curvature of metric (2.6) in string units is proportional, up to a bounded factor, to the curvature in string units of the background with $B = 0$.

It is convenient to define dimensionless effective non commutativity parameters

$$a_i^{eff} = a_i u \sim \left(\frac{b_i u^2}{g_{eff}}\right)^{\frac{2}{7-p}}, \qquad i = 1, 3, \ldots, 2m-1\,. \tag{2.10}$$

At large distances $L \gg \sqrt{b_i}/g_{eff}$ we have $a_i^{eff} \ll 1$, the noncommutative effects are small and the effective description of the worldvolume theory is in terms of a commutative field theory. In this regime the supergravity solutions (2.6) reduce to the low energy backgrounds considered in [16]. The noncommutativity of the worldvolume theory is relevant at distances $L \leq \sqrt{b_i}/g_{eff}$ where $a_i^{eff} \geq 1$. The noncommutativity effects can be neglected at energies

$$u \ll \left(\bar{g}_{YM}^2 N b_i^{-1} \prod_{j \neq i}^{2m-1} b_j\right)^{\frac{1}{7-p}}, \qquad i = 1, 3, \ldots, 2m-1\,. \tag{2.11}$$

The effective string coupling e^{ϕ} in (2.6) reads

$$e^{\phi} \sim \frac{g_{eff}^{\frac{7-p}{2}}}{N \prod_{i\ \mathrm{odd}}^{2m-1} (1 + (a_i^{eff})^{7-p})^{1/2}}\,. \tag{2.12}$$

Keeping g_{eff} and a_i^{eff} fixed we see from (2.12) that $e^{\phi} \sim 1/N$. Thus the string loop expansion corresponds to the $1/N$ expansion of the noncommutative gauge theory. Note also that at large u (UV) the dilaton in (2.6) reads

$$e^{\phi} \sim u^{(7-p)(p-2m-3)/4}, \tag{2.13}$$

which blows up for $p > 2m+3$. At small u (IR) the dilaton blows up for $p < 3$ independently of the B field.

We define two scales which will be useful for the discussion in the following sections. One scale is the energy scale where the effective string coupling is of order one while the noncommutative effects are negligible. It reads

$$u \sim \left(\frac{N^{\frac{p-3}{7-p}}}{\bar{g}_{YM}^2 \prod_{i\ \mathrm{odd}}^{2m-1} b_i}\right)^{\frac{1}{p-3}}. \tag{2.14}$$

The second scale is the energy scale where the effective string coupling is of order one while the noncommutative effects are large $a_i^{eff} \gg 1$. It reads

$$u \sim \left(\bar{g}_{YM}^{\frac{14-2p+4m}{3-p+2m}} N \prod_{i\ \mathrm{odd}}^{2m-1} b_i\right)^{\frac{1}{7-p}}. \tag{2.15}$$

Finally, the supergravity action with the background (2.6)

$$l_s^{-8}\int\sqrt{-g}e^{-2\phi}\mathcal{R}\sim N^{\frac{p+1}{2}}\,, \tag{2.16}$$

as for the $B=0$, suggesting that the number of degrees of freedom at large N is the same for the noncommutative and commutative field theories [12].

2.3 Phase diagrams

Summarizing the above discussion, the effective dimensionless expansion parameters of the Dp-branes system in the background of non-zero B fields are the number of branes N, the effective gauge coupling g_{eff} and the effective noncommutativity parameters $a_i^{eff}, i=1,3,\ldots,2m-1$. For each D$p$-brane we can plot a phase diagram as a function of these dimensionless parameters. Different regions of these phase diagrams will have a good description in terms of different variables. Such analysis when $B=0$ was done in [16].

D2 branes. Consider the supergravity solution of N D2-branes in the presence of B field (2.6). In this case $m=1$, only the B_{12} component is non-zero. Thus,

$$\begin{aligned}
ds^2 &= l_s^2\left[\frac{u^{5/2}}{R^{5/2}}\left(-dt^2+\frac{dx_1^2+dx_2^2}{1+a^5u^5}\right)+\frac{R^{5/2}}{u^{5/2}}(du^2+u^2d\Omega_6^2)\right],\\
B_{12} &= \frac{l_s^2}{b}\frac{a^5u^5}{1+a^5u^5}\,,\\
e^{2\phi} &\sim \frac{(\bar{g}_{YM}^{10}Nb^5)^{1/2}}{u^{5/2}(1+a^5u^5)}\,,
\end{aligned} \tag{2.17}$$

where B_{12} is the B field scaled in accord with the coordinates rescaling. When $a^{eff}\ll 1$ the noncommutativity effects are small and we have a good description in terms of a commutative field theory. This is valid at low energies $u\ll(\bar{g}_{YM}^2N/b)^{1/5}$.

Consider the flow from high energies to low energies. The effective dimensionless coupling (2.9) is now $g_{eff}^2\sim\bar{g}_{YM}^2Nb/u$. When $g_{eff}\ll 1$ we have a good description in terms of noncommutative $\mathcal{N}=8$ perturbative noncommutative super Yang-Mills (NCSYM). The energy range for this description to be valid is $u\gg\bar{g}_{YM}^2Nb$. When $g_{eff}\sim 1$, that is $u\sim\bar{g}_{YM}^2Nb$, we have a transition to the type IIA supergravity description. The type IIA supergravity description is valid when both the curvature in string units (2.8) and the effective string coupling (2.12) are small. This implies large N (or large noncommutativity parameter a^{eff}). When the effective string coupling is large the good description is in terms of an eleven dimensional theory. This description is obtained by uplifting the D2 brane solution (2.17) to eleven dimensions. When uplifting to eleven dimensions we can distinguish two cases. In the first case the effective string coupling becomes large before the noncommutative effects can be neglected while in the second case it becomes large after the noncommutative effects

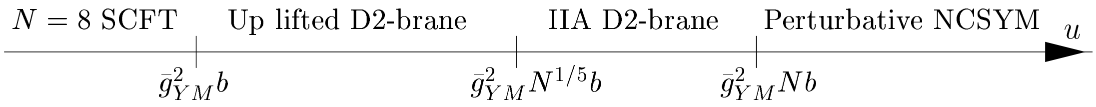

Figure 1: The different descriptions of the D2 branes theory with non-zero B field as a function of the energy scale u. We see the flow from $\mathcal{N} = 8$ NCSYM at high energy to $\mathcal{N} = 8$ SCFT at low energy. The plot is for the case $\beta \ll 1$ and therefor when we up-lift to eleven dimensions the noncommutativity effects are negligible. When $\beta \gg 1$ the plot is similar, however the transition to eleven dimensions occurs at $u \sim \bar{g}_{YM}^{14/15} N^{1/5} b^{1/5}$ and then the noncommutative effects are not negligible.

become negligible. It is convenient to define a dimensionless parameter β which is the ratio between the energy scale at which the effective string coupling is of order one while the noncommutative effects are negligible and the energy scale at which the dimensionless noncommutative parameter a^{eff} is of order one. It reads $\beta = \bar{g}^4_{YM} b^3$. Then the first case corresponds to $\beta \gg 1$ and the second case to $\beta \ll 1$. Finally, at energies $u \ll \bar{g}^2_{YM} b$ the good description is in terms of the eleven dimensional M2 branes background. In figure 1 we plot the transition between the different descriptions as a function of the energy scale u. We see the flow from $\mathcal{N} = 8$ NCSYM at high energy to $\mathcal{N} = 8$ SCFT at low energy.

D4 branes. We will consider now N D4 branes in the presence of a non-zero B field. The rank $2m$ of the B field can be two or four.

The case $m = 1$. When $a^{eff} \ll 1$ the noncommutativity effects are small and we have a good description in terms of a commutative field theory. This is valid at low energies $u \ll (\bar{g}^2_{YM} N/b)^{1/3}$. Consider the flow from low energies to high energies. The effective dimensionless coupling (2.9) is now $g^2_{eff} \sim \bar{g}^2_{YM} N b u$. When $g_{eff} \ll 1$ we have a good description in terms of a maximally supersymmetric five dimensional Yang-Mills theory. The energy range for this description to be valid is $u \ll 1/\bar{g}^2_{YM} N b$. When $g_{eff} \sim 1$, that is $u \sim 1/\bar{g}^2_{YM} N b$ we have a transition to the type IIA supergravity description. The type IIA supergravity description is valid when both the curvature in string units (2.8) and the effective string coupling (2.12) are small. This implies large N or large noncommutativity parameter a^{eff}. When the effective string coupling is large the good description is in terms of an eleven dimensional theory. This description is obtained by uplifting the D4 brane solution to eleven dimensions. As in the D2 brane case, when uplifting to eleven dimensions we can distinguish two cases. In the first case the effective string coupling becomes large before the noncommutative effects become significant while in the second case it becomes large after the noncommutative effects become significant. The the ratio between the energy scale at which the effective string coupling is of order one and the energy scale at which the dimensionless noncommutative parameter a^{eff} is of order one reads now $\beta = 1/\bar{g}^4_{YM} b$. The first case corresponds to $\beta \ll 1$ and the second case

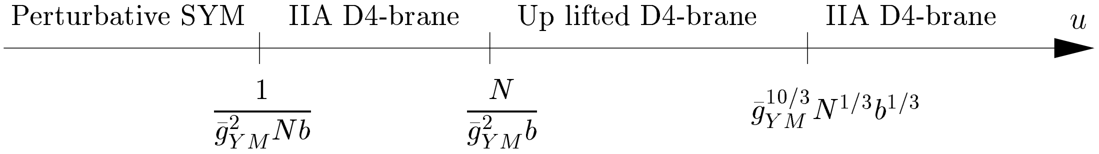

Figure 2: The different descriptions of the D4 branes theory with B field ($m = 1$) as a function of the energy scale u for $\beta \ll 1$.

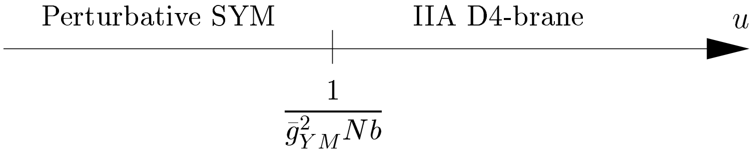

Figure 3: The different descriptions of the D4 branes theory with B field ($m = 1$) as a function of the energy scale u for $\beta \gg 1$.

to $\beta \gg 1$. When $\beta \ll 1$ we up lift to eleven dimensions at energy $u \sim N^{1/3}/\bar{g}_{YM}^2 b$. As we increase the energy the noncommutative effects become large and the effective string coupling decreases. It becomes small again at energies $u \gg \bar{g}_{YM}^{10/3} N^{1/3} b^{1/3}$ and we have a good description by the type IIA supergravity background. In figure 2 we plot the transition between the different descriptions as a function of the energy scale u. Finally, when $\beta \gg 1$ we do not have to up lift to eleven dimensions. The reason being that the effective string coupling is kept small by the large noncommutative effects. This is described in figure 3.

The case $m = 2$. The case $m = 2$ is similar to to the $m = 1$ case and we will briefly discuss it. For simplicity consider the case $b_1 = b_3 = b$. It is again convenient to define the dimensionless parameter β which now reads $\beta = 1/\bar{g}_{YM}^4 b^3$. The phase diagram for the cases $\beta \gg 1$ and $\beta \ll 1$ are similar to the $m = 1$ case above. The energy scales at which the transitions occur are, of course, modified.

D5 branes. Consider now the theory of N D5 branes of type IIB string theory in the presence of a B field. The rank of the B-field can be up to six, $m = 1, 2, 3$.

The case $m = 1$. The noncommutativity effects are small and we have a good description in terms of a commutative field theory at low energies $u \ll (\bar{g}_{YM}^2 N/b)^{1/2}$. Consider the flow from low energies to high energies. The effective dimensionless coupling (2.9) is now $g_{eff}^2 \sim \bar{g}_{YM}^2 N b u^2$. When $g_{eff} \ll 1$ we have a good description in terms of a maximally supersymmetric six dimensional Yang-Mills theory. The energy range for this description to be valid is $u \ll (1/\bar{g}_{YM}^2 N b)^{1/2}$. When $g_{eff} \sim 1$, that is $u \sim (1/\bar{g}_{YM}^2 N b)^{1/2}$ we have a transition to the type IIB supergravity description. The type IIB supergravity description is valid when both the curvature in string units (2.8) and the effective string coupling (2.12) are small. As before, this implies large N or large noncommutativity parameter a^{eff}. When effective string coupling is large the good description is in terms of an S-dual ten dimensional theory. We

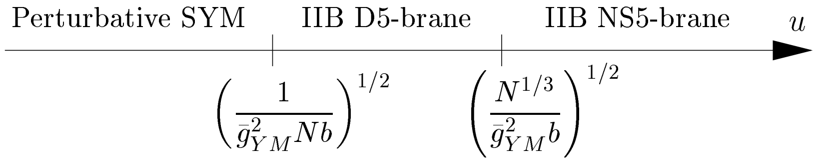

Figure 4: The transition between the different descriptions of the D5 brane theory with B field ($m = 1$) as a function of the energy scale u when $\beta \ll 1$.

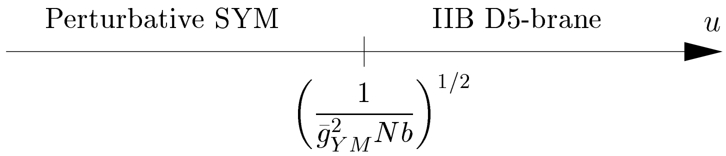

Figure 5: The transition between the different descriptions of the D5 brane theory with B field ($m = 1$) as a function of the energy scale u when $\beta \gg 1$.

distinguish two cases. In the first case the effective string coupling becomes large before the noncommutative effects become significant while in the second case it becomes large after the noncommutative effects become significant. The the ratio between the energy scale at which the effective string coupling is of order one and the energy scale at which the dimensionless noncommutative parameter a^{eff} is of order one reads now $\beta = 1/\bar{g}_{YM}^2$. The first case corresponds to $\beta \ll 1$ and the second case to $\beta \gg 1$. When $\beta \ll 1$ we use the S-dual description when $u \sim (N/\bar{g}_{YM}^2 b)^{1/2}$. As we increase the energy the noncommutative effects become large and the effective string coupling approaches the value $1/\beta = \bar{g}_{YM}^2$. In figure 4 we plot the transition between the different descriptions as a function of the energy scale u. When $\beta \gg 1$ the effective string coupling is kept small by the large noncommutative effects and we do not need the S-dual description. This is described in figure 5.

The case $m = 2$**.** For a simplicity of the discussion we will assume $b_1 = b_3 = b$. The noncommutativity effects are small and we have a good description in terms of a commutative field theory at low energies $u \ll (\bar{g}_{YM}^2 N)^{1/2}$. Consider the flow from low energies to high energies. The effective dimensionless coupling (2.9) is now $g_{eff}^2 \sim \bar{g}_{YM}^2 N b^2 u^2$. When $g_{eff} \ll 1$ we have a good description in terms of a maximally supersymmetric six dimensional Yang-Mills theory. The energy range for this description to be valid is $u \ll (1/\bar{g}_{YM}^2 N b^2)^{1/2}$. When $g_{eff} \sim 1$, that is $u \sim (1/\bar{g}_{YM}^2 N b^2)^{1/2}$ we have a transition to the type IIB supergravity description. When the effective string coupling is large we have to pass to the S-dual description. As in the previous analysis, we distinguish two cases. In the first case the effective string coupling becomes large before the noncommutative effects become significant while in the second case it becomes large after the noncommutative effects become significant. The the ratio between the energy scale at which the effective string coupling is of order one and the energy scale at which the dimensionless noncommutative parameter a^{eff} is of order one reads now $\beta = 1/\bar{g}_{YM}^2 b$. The first case corresponds to $\beta \ll 1$

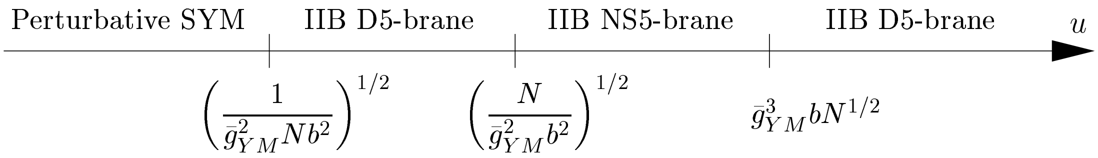

Figure 6: The transition between the different descriptions of the D5 brane theory with B field ($m = 2$) as a function of the energy scale u when $\beta \ll 1$.

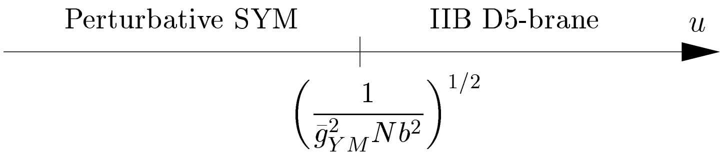

Figure 7: The transition between the different descriptions of the D5 brane theory with B field ($m = 2$) as a function of the energy scale u when $\beta \gg 1$.

and the second case to $\beta \gg 1$. When $\beta \ll 1$ we use the S-dual description when $u \sim (N/\bar{g}_{YM}^2 b^2)^{1/2}$. As we increase the energy the noncommutative effects become large and the effective string coupling decreases. At energy scales $u \gg \bar{g}_{YM}^3 b N^{1/2}$ we can use the type IIB description again. In figure 6 we plot the transition between the different descriptions as a function of the energy scale u. When $\beta \gg 1$ the effective string coupling is kept small by the large noncommutative effects and we do not need the S-dual description. This is described in figure 7.

The case $m = 3$. The case of $m = 3$ is similar to to the $m = 2$ case and we will briefly discuss it. We consider the euclidean signature and again assume $b_1 = b_3 = b_5 = b$. It is again convenient to define the dimensionless parameter β which now reads $\beta = 1/\bar{g}_{YM}^2 b$. The phase diagram for the cases $\beta \gg 1$ and $\beta \ll 1$ are similar to the $m = 2$ case above. The energy scales at which the transitions occur are modified.

D6 branes. With a vanishing B field the worldvolume theory of N D6 branes of type IIA string theory does not decouple from the bulk. This can be seen, for instance, by the fact that in the decoupling limit we keep $g_{YM}^2 = g_s l_s^3 = fixed$ as $l_s \to 0$. This means that the eleven dimensional Planck length $l_p = g_s^{1/3} l_s$ is kept fixed and that gravity does not decouple.

Consider now N D6 branes of type IIA in the presence of a B field. In this case the rank of the B field can be up to six, $m = 1, 2, 3$. The effective string coupling (2.13) at large u reads $e^{\phi} \sim u^{(3-2m)/4}$. When $m = 1$ we expect to have an eleven dimensional description in the UV. Note that in the decoupling limit we keep $g_s l_s^{3-2m}$ =fixed as $l_s \to 0$. Therefor for $m = 1$ the the eleven dimensional Planck length $l_p \to 0$ and we expect gravity to decouple. For $m = 2, 3$ the effective string coupling is small at all energy scales and there is no need for an eleven dimensional description at high energy. The ten dimensional Planck scale $l_p^{(10)} = g_s^{1/4} l_s \to 0$ and we expect gravity to decouple. In the following we will analyse the phase diagram of the D6 branes system.

The background in the limit (2.5) takes the form

$$
\begin{aligned}
l_s^{-2} ds^2 &= \frac{u^{1/2}}{R^{1/2}} \left(\sum_{i \text{ odd}}^{2m-1} h_i (dx_i^2 + dx_{i+1}^2) + \cdots + dx_7^2 \right) + \frac{R^{1/2}}{u^{1/2}} (du^2 + u^2 d\Omega_2^2) \,, \\
R &= c_p \bar{g}_{YM}^2 N \prod_{i \text{ odd}}^{2m-1} b_i \,, \qquad\qquad a_i = \frac{b_i^2}{R} \,, \\
B_{i,i+1} &= \frac{l_s^2}{b_i} \frac{a_i u}{1 + a_i u} \,, \qquad\qquad h_i = \frac{1}{1 + a_i u} \,, \\
e^{2\phi} &\sim \left(\frac{\bar{g}_{YM}^2 \prod_i b_i}{N^3} \right)^{1/2} u^{3/2} \prod_{i \text{ odd}}^{2m-1} \frac{1}{1 + a_i u} \,.
\end{aligned}
\tag{2.18}
$$

The case $m = 1$. The noncommutativity effects are small and we have a good description in terms of a commutative field theory at low energies $u \ll \bar{g}_{YM}^2 N/b$. Consider the flow from low energies to high energies. The effective dimensionless coupling (2.9) is now $g_{eff}^2 \sim \bar{g}_{YM}^2 N b u^3$. When $g_{eff} \ll 1$ we have a good description in terms of a perturbative maximally supersymmetric seven dimensional Yang-Mills theory. The energy range for this description to be valid is $u \ll (1/\bar{g}_{YM}^2 N b)^{1/3}$. When $g_{eff} \sim 1$, that is $u \sim (1/\bar{g}_{YM}^2 N b)^{1/3}$ we have a transition to the type IIA supergravity description. When effective string coupling is large the good description is in terms of an eleven dimensional theory. As before, we distinguish two cases. In the first case the effective string coupling becomes large before the noncommutative effects become significant while in the second case it becomes large after the noncommutative effects become significant. The the ratio between the energy scale at which the effective string coupling is of order one and the energy scale at which the dimensionless noncommutative parameter a^{eff} is of order one reads now $\beta = b/\bar{g}_{YM}^4$. The first case corresponds to $\beta \ll 1$ and the second case to $\beta \gg 1$. When $\beta \ll 1$ we use the eleven dimensional supergravity description when $u \sim N/(\bar{g}_{YM}^2 b)^{1/3}$. The eleven dimensional curvature is small for every N when $u > N/(\bar{g}_{YM}^2 b)^{1/3}$

$$
l_p^2 \mathcal{R}_{11} \sim e^{2\phi/3} \frac{1}{g_{eff}} < \frac{1}{N^2} \,, \tag{2.19}
$$

and vanishes for $u \gg N/(\bar{g}_{YM}^2 b)^{1/3}$.Thus, similar to the case without a B field [16], the eleven dimensional supergravity solution can be trusted in the UV for any N. Unlike the $B = 0$ case, the metric at large u is not the flat eleven dimensional one. As we discussed above, since the eleven dimensional Planck length goes to zero in the decoupling limit we expect gravity to decouple from the branes worldvolume theory. Thus, it is plausible that a seven dimensional worldvolume theory without gravity does exist.

When $\beta \gg 1$ the phase diagram is similar, however the transition to eleven dimensions occurs at $u \sim Nb/\bar{g}_{YM}^6$ and then the noncommutative effects are not

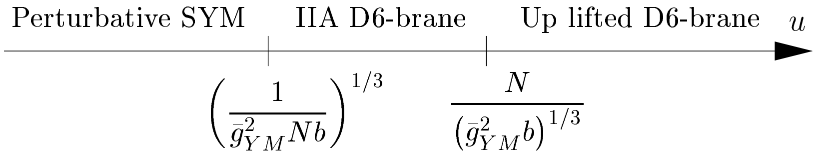

Figure 8: The transition between the different descriptions of the D6 brane theory with B field ($m = 1$) as a function of the energy scale u when $\beta \ll 1$. When $\beta \gg 1$ the plot is similar. However, the transition to the eleven dimensional description is at $u \sim Nb/\bar{g}_{YM}^6$.

negligible. Similarly, the eleven dimensional supergravity solution can be trusted in the UV for any N. In figure 8 we plot the transition between the different descriptions as a function of the energy scale u.

The case $m = 2$. For a simplicity of the discussion we will assume $b_1 = b_3 = b$. The noncommutativity effects are small and we have a good description in terms of a commutative field theory at low energies $u \ll \bar{g}_{YM}^2 N$. Consider the flow from low energies to high energies. The effective dimensionless coupling (2.9) is now $g_{eff}^2 \sim \bar{g}_{YM}^2 N b^2 u^3$. When $g_{eff} \ll 1$ we have a good description in terms of perturbative seven dimensional Yang-Mills theory. The energy range for this description to be valid is $u \ll (1/\bar{g}_{YM}^2 N b^2)^{1/3}$. When $g_{eff} \sim 1$, that is $u \sim (1/\bar{g}_{YM}^2 N b^2)^{1/3}$ we have a transition to the type IIA supergravity description. As in the previous analysis, we distinguish two cases. In the first case the effective string coupling becomes large before the noncommutative effects become significant while in the second case it becomes large after the noncommutative effects become significant. The the ratio between the energy scale at which the effective string coupling is of order one and the energy scale at which the dimensionless noncommutative parameter a^{eff} is of order one reads now $\beta = 1/\bar{g}_{YM}^4 b$. The first case corresponds to $\beta \ll 1$ and the second case to $\beta \gg 1$. When $\beta \ll 1$ we use the eleven dimensional description when $u \sim N/(\bar{g}_{YM}^2 b^2)^{1/3}$. As we increase the energy the noncommutative effects become large and the effective string coupling decreases. At energy scales $u \gg \bar{g}_{YM}^{10} N b^2$ we can use the type IIA description again. The ten dimensional curvature is small for every N when $u > \bar{g}_{YM}^{10} N b^2$

$$l_s^2 \mathcal{R}_{10} < \frac{1}{N^2}\,, \tag{2.20}$$

and vanishes for $u \gg \bar{g}_{YM}^{10} N b^2$.Thus, the ten dimensional supergravity solution can be trusted in the UV for any N. Note, however, the metric at large u is not flat. In figure 9 we plot the transition between the different descriptions as a function of the energy scale u.

When $\beta \gg 1$ the effective string coupling is kept small by the large noncommutative effects and we do not need the eleven dimensional description. The ten dimensional curvature is small for every N when $u > N/(\bar{g}_{YM}^2 b^2)^{1/3}$ and the ten dimensional supergravity solution can be trusted in the UV for any N. This is described in figure 10. The interaction lagrangean between the brane modes and the

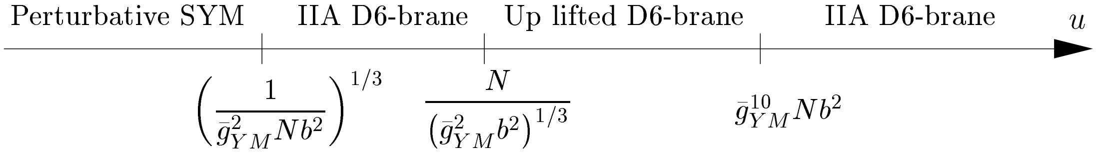

Figure 9: The transition between the different descriptions of the D6 brane theory with B field ($m = 2$) as a function of the energy scale u when $\beta \ll 1$.

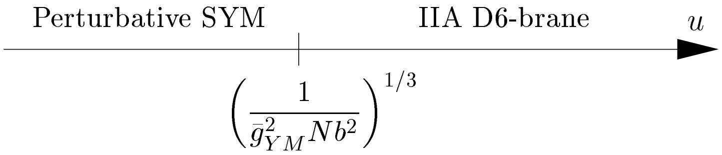

Figure 10: The transition between the different descriptions of the D6 brane theory with B field ($m = 2$) as a function of the energy scale u when $\beta \gg 1$.

bulk modes is proportional to positive powers of $\kappa_{10} = g_s l_s^4$ which goes to zero in the decoupling limit. Thus we expect all the interaction terms to vanish in this limit and gravity to decouple from the branes worldvolume theory. Thus, it is plausible to expect that a seven dimensional worldvolume theory without gravity does exist. It was noted in [8] that such a theory will have a negative specific heat.

The case $m = 3$. The case $m = 3$ is similar to to the $m = 2$ case and we will briefly discuss it. We assume $b_1 = b_3 = b_5 = b$. It is again convenient to define the dimensionless parameter β which now reads $\beta = 1/\bar{g}_{YM}^4 b^3$. The phase diagram for the cases $\beta \gg 1$ and $\beta \ll 1$ are similar to the $m = 2$ case above. The energy scales at which the transitions occur are modified. As for the $m = 2$ case, the scalar curvature vanishes at large u, however, the metric at large u is not flat. The decoupling from the bulk argument is as in the $m = 2$ case.

Dp-branes ($p > 6$) Consider now the decoupling limit for Dp-branes with $p > 6$. In a ten dimensional description the interaction lagrangean between the brane modes and gravity is proportional to positive powers of $\kappa_{10} = g_s l_s^4$. In an eleven dimensional description the interaction is proportional to positive powers of the eleven dimensional Planck length l_p. Consider first the D7 branes. In the decoupling limit we hold $g_s l_s^{4-2m}$ fixed as $l_s \to 0$. Therefor, when $m > 0$ we see that $\kappa_{10} \to 0$ in this limit, which indicates that the worldvolume theory decouples from gravity. In the D8 branes case we hold $g_s l_s^{5-2m}$ fixed as $l_s \to 0$. Again, when $m > 0$ we see that $\kappa_{10} \to 0$ in this limit. When $m = 1$ the effective string coupling is small in the UV and the ten dimensional description is sufficient. When $m > 1$ the effective string coupling is large in the UV and we will need an eleven dimensional description. Note that $l_p \to 0$ when $m > 1$ which indicates that again gravity decouples from the brane worldvolume theory. For D9 branes we hold $g_s l_s^{4-2m}$ fixed as $l_s \to 0$ which ensures again that $\kappa_{10} \to 0$ and indicates the decoupling of gravity.

2.4 Non extremal Dp-branes

Consider the non-extremal Dp-branes solution with non zero B field. The metric in the decoupling limit reads:

$$
\begin{aligned}
l_s^{-2}ds^2 = &\left(\frac{u}{R}\right)^{\frac{7-p}{2}} \left(\sum_{i \text{ odd}}^{2m-1} h_i(dx_i^2 + dx_{i+1}^2) + \cdots + \left(1-\left(\frac{u_T}{u}\right)^{7-p}\right) dx_{p+1}^2\right) + \\
&+ \left(\frac{R}{u}\right)^{\frac{7-p}{2}} \left(\frac{du^2}{1-(u_T/u)^{7-p}} + u^2 d\Omega_{8-p}^2\right),
\end{aligned}
\tag{2.21}
$$

where u_T is related to the energy density of the brane above density extremality ε by

$$
u_T^{7-p} \sim \bar{g}_{YM}^4 \prod_{i \text{ odd}}^{2m-1} b_i^2 \varepsilon \,. \tag{2.22}
$$

This should correspond to decoupled theories at finite temperature with ε being the energy density of the field theory. As discussed in [8] the thermodynamic quantities are as in the case without a B field. More precisely, they are the same as in the $B = 0$ case with $g_{YM}^2 \to \bar{g}_{YM}^2 \prod_{i \text{ odd}}^{2m-1} b_i$. Later we will analyse the Wilson loops of this system.

3. Fivebranes

In this section we will discuss possible noncommutative deformations of the M5 branes and NS5 branes worldvolume theories.

3.1 M5 branes

Consider N coinciding M5 branes in the presence of a nonzero C field with $m = 1, 2$. The supergravity solution reads[4]

$$
\begin{aligned}
ds_{11}^2 &= f^{-1/3}\left[\left(\prod_{i \text{ odd}}^{2m-1} h_i\right)^{-1/3} \left(\sum_{i \text{ odd}}^{2m-1} h_i(dx_i^2 + dx_{i+1}^2) + dx_6^2\right) + \left(\prod_{i \text{ odd}}^{2m-1} h_i\right)^{2/3} dx_5^2\right] + \\
&\quad + f^{2/3}\left(\prod_{i \text{ odd}}^{2m-1} h_i\right)^{-1/3} (dr^2 + r^2 d\Omega_4^2)\,, \\
f &= 1 + \frac{\pi N l_p^3}{\prod_i \cos\theta_i r^3}\,, \\
h_i^{-1} &= \sin^2\theta_i f^{-1} + \cos^2\theta_i\,, \\
C_{5,i,i+1} &= \tan\theta_i\, f^{-1}\, h_i\,, \qquad C_{i,i+1,6} = \sin\theta_{4-i}\cos\theta_i\, f^{-1}\, h_i\,.
\end{aligned}
\tag{3.1}
$$

[4] In the following we will not write the component of the C field associated with the M5 branes charge.

Let us discuss first the case when the worldvolume coordinate x_5 is compactified on a circle of radius R_0. In the decoupling limit we send $l_p \to 0$ and keep following quantities fixed:

$$\begin{aligned} u &= \frac{r}{l_p^3} R_0 \,, & \bar{R}_0 &= \frac{R_0}{l_p^{3m/(m+1)}} \,, \\ b_i &= \frac{l_p^3}{R_0} \tan\theta_i & \bar{x}_{i,i+1} &= \frac{R_0 b_i}{l_p^3} x_{i,i+1} \\ \bar{x}_6 &= x_6 & \bar{x}_5 &= \frac{\prod_i b_i R_0^m}{l_p^{3m}} x_5 \,. \end{aligned} \tag{3.2}$$

This decoupling limit is consistent with the D4 branes decoupling limit where we use the relation $l_s^2 R_0 = l_p^3$. The same scaling of the coordinates x was derived in [5] for the case of $m = 2$ and $b_i = b$.

In the limit (3.2), the supergravity solution reads:

$$\begin{aligned} l_p^{-2} ds_{11}^2 &= \left(\prod_i h_i\right)^{1/3} \frac{u}{(\pi N)^{1/3} \prod_i b_i \bar{R}_0^{m+1}} \left(\sum_i h_i^{-1}(dx_i^2 + dx_{i+1}^2) + dx_6^2 + \prod_i h_i^{-1} dx_5^2\right) + \\ &+ \left(\prod_i h_i\right)^{1/3} \frac{(\pi N)^{2/3}}{u^2}(du^2 + u^2 d\Omega_4^2) \,, \\ h_i &= 1 + a_i^3 u^3 \,, \qquad a_i^3 = \frac{b_i^2}{\pi N \bar{R}_0^{m+1} \prod_j b_j} \,. \end{aligned} \tag{3.3}$$

The C field (up to numerical factors) takes the form

$$\begin{aligned} m = 1: &\qquad C_{346} \sim \frac{l_p^3}{b^2 \bar{R}_0^2} a^3 u^3 \,, & C_{125} &\sim \frac{l_p^3}{b^2 \bar{R}_0^2} \frac{a^3 u^3}{1 + a^3 u^3} \,, \\ m = 2: &\qquad C_{i,i+1,6} \sim \frac{l_p^3}{b_i^3 \bar{R}_0^3} \frac{a_i^3 u^3}{1 + a_i^3 u^3} \,, & C_{i,i+1,5} &\sim \frac{l_p^3}{b_i \prod_j b_j \bar{R}_0^3} \frac{a_i^3 u^3}{1 + a_i^3 u^3} \,. \end{aligned} \tag{3.4}$$

This background is the ten dimensional D4 branes solution up lifted to eleven dimensions. At low energies compared to $1/R_0$ the description of the system is in terms of the D4 brane theory, as discussed in the previous section. We might have expected that at high energies (large u) we will have a good description in terms of a noncommutative $(0, 2)$ theory in six dimensions. The curvature reads

$$l_p^2 \mathcal{R}_{11} \sim \frac{1}{N^{2/3} \prod_{i \text{ odd}}^{2m-1} (1 + a_i^3 u^3)^{1/3}} \,, \tag{3.5}$$

and we can trust the supergravity solution. However, the size of the compact direction, x_5, is controlled $\prod_{i \text{ odd}}^{2m-1} h_i^{-2/3} u / N^{1/3}$ which decreases in the UV. Therefor, at large u we are back in the ten dimensional D4 branes background, as discussed in the previous section and we do not find a six dimensional field theory description of the UV.

Let us discuss now the M5 branes in the background of a nonzero C field without wrapping a circle. Consider the supergravity solution (3.1) and let us keep the following quantities fixed as $l_p \to 0$

$$u^{n-1} = \frac{r}{l_p^n}, \qquad b_i^{q/2} = l_p^q \tan\theta_i\,. \tag{3.6}$$

For the moment we will consider $n > 1, q$ as arbitrary positive integers. We get

$$\begin{aligned} f &= 1 + \frac{\pi N \prod_i b_i^{q/2}}{l_p^{mq+3n-3} u^{3n-3}}\,, \\ h_i &= \frac{b_i^q}{l_p^{2q}} \frac{1}{1 + \frac{b_i^q}{l_p^{2q}}\left(1 + \frac{\pi N \prod_i b_i^{q/2}}{l_p^{mq+3n-3} u^{3n-3}}\right)^{-1}}\,. \end{aligned} \tag{3.7}$$

The condition for a finite metric solution and a constant nonzero C field at infinity require $(m-2)q \le 3(1-n) < mq$. Keeping finite the tension of the strings that arise from M2 branes stretched between the M5 branes requires $n = 3$, namely $u^2 = r/l_p^3$ =fixed. This implies $m = 1$, $q = 6$. The background reads

$$\begin{aligned} l_p^{-2} ds_{11}^2 &= \frac{u^2}{(\pi N)^{1/3}} h^{1/3}(h^{-1}dx_{1,2,5}^2 + dx_{3,4,6}^2) + \frac{(\pi N)^{2/3}}{u^2} h^{1/3}(4du^2 + u^2 d\Omega_4^2)\,, \\ h &= 1 + a^6 u^6\,, \\ C_{346} &= \frac{l_p^3}{b^{3/2}} a^6 u^6\,, \qquad C_{125} = \frac{l_p^3}{b^{3/2}} \frac{a^6 u^6}{1 + a^6 u^6}\,, \end{aligned} \tag{3.8}$$

where $a^6 = b^3/\pi N$ and we rescaled the coordinates $x_{3,4,6} \to l_p^3/b^{3/2} x_{3,4,6}$ and $x_{1,2,3} \to b^{3/2}/l_p^3 x_{1,2,3}$. Note that the decoupling limit leading to (3.8) differs from (3.2).

At very low energies (small u) the metric (3.8) describes the eleven dimensional $AdS_7 \times S^4$ background with a self-dual C field, providing a dual description of the $(0,2)$ SCFT. As we increase u the $AdS_7 \times S^4$ is deformed and the C field is no longer self-dual. The curvature reads

$$l_p^2 \mathcal{R}_{11} \sim \frac{1}{N^{2/3}(1 + a^6 u^6)^{1/3}}\,, \tag{3.9}$$

and we can trust the supergravity solution in the UV as well.

3.2 NS5 branes

Type IIB. The type IIB NS5 branes solution in the presence of nonzero RR fields can be obtained from D5 branes by S-duality transformation. Under S-duality we have:

$$\begin{aligned} l_s^2 &\to l'^2_s \equiv g_s l_s^2\,, \qquad & g_s &\to g'_s \equiv \frac{1}{g_s}\,, \\ e^{\phi} &\to e^{\phi'} \equiv e^{-\phi}\,, \qquad & ds^2 &\to ds'^2 \equiv g_s e^{-\phi} ds^2\,. \end{aligned} \tag{3.10}$$

Using (3.10) we get the type IIB NS5 branes background,

$$
\begin{aligned}
ds'^2 &= \prod_{i\ \mathrm{odd}}^{2m-1} h_i^{-1/2}\left[\sum_{i\ \mathrm{odd}}^{2m-1} h_i(dx_i^2+dx_{i+1}^2)+\cdots+dx_6^2+f(dr^2+r^2d\Omega_3^2)\right],\\
f &= 1+\frac{c_5Nl'^2_s}{\prod_{i\ \mathrm{odd}}^{2m-1}\cos\theta_i\, r^2},\\
h_i^{-1} &= \sin^2\theta_i f^{-1}+\cos^2\theta_i\,,\\
e^{2\phi'} &= g'^2_s f\prod_{i\ \mathrm{odd}}^{2m-1} h_i^{-1}\,,
\end{aligned}
\tag{3.11}
$$

and the NS field B_{ij} is mapped to a the RR field A_{ij}. The decoupling limit is derived by applying (3.10) on the decoupling limit of the D5 branes. It is defined by taking the limit $g'_s l'^2_s \to 0$ and keeping fixed

$$
\begin{aligned}
u &= \frac{r}{g'_s l'^2_s}\,, & \bar{g}'_s &= g'^{-m}_s l'^{2-2m}_s\\
b_i &= g'_s l'^2_s\tan\theta_i\,, & \bar{x}_{i,i+1} &= \frac{b_i}{g'_s l'^2_s}x_{i,i+1}\,.
\end{aligned}
\tag{3.12}
$$

Keeping u fixed means keeping fixed the mass of a D-string stretched between two NS5 branes.

In the limit (3.12) the background (3.11) reads

$$
\begin{aligned}
ds'^2 &= \frac{l'^2_s}{\bar{g}'_s\prod_i b_i}\prod_{i\ \mathrm{odd}}^{2m-1} h_i^{1/2}\left[\sum_{i\ \mathrm{odd}}^{2m-1} h_i^{-1}(dx_i^2+dx_{i+1}^2)+\cdots\right.\\
&\qquad\left.+dx_6^2+\frac{c_5N\bar{g}'_s}{u^2}\prod_{i\ \mathrm{odd}}^{2m-1} b_i(du^2+u^2d\Omega_3^2)\right],\\
h_i &= 1+a_i^2u^2\,,\\
a_i^2 &= \frac{b_i^2}{c_5N\prod_{j\ \mathrm{odd}}^{2m-1} b_j\bar{g}'_s}\,,\\
e^{2\phi'} &= \frac{c_5N}{\prod_{i\ \mathrm{odd}}^{2m-1} b_i\bar{g}'_s u^2}\prod_{i\ \mathrm{odd}}^{2m-1} h_i\,.
\end{aligned}
\tag{3.13}
$$

The Yang-Mills coupling of the worldvolume theory is $g^2_{YM}\sim g'^{-m}_s l'^{2-2m}_s$. The curvature of the metric reads

$$
l'^2_s\mathcal{R}\sim\frac{1}{N}\frac{1}{\prod_i(1+a_i^2u^2)^{1/2}}\,.
\tag{3.14}
$$

When $a_i^{eff}\equiv a_iu\ll 1$ the supergravity approximation can be trusted for large N, while when $a_i^{eff}\gg 1$ the supergravity approximation can be trusted for finite N. When $m=1$ we see that at large u the effective string coupling is small and we can use the NS5 brane description in the UV. When $m = 2, 3$ the effective string coupling is large in the UV and we have to use the S-dual description of D5 branes. This is precisely what we saw in the phase structure of the D5 branes system in the previous section.

Type IIA The background of type IIA NS5 branes wrapping a circle can be obtained by a T-duality transformation [17] of the type IIB NS5 branes. We compactify the coordinate x_5 on a circle and perform T-duality in x_5 on the background (3.13). The decoupled type IIA NS5 branes solution reads

$$
\begin{aligned}
ds'^2 &= \frac{l'^2_s}{\bar{g}'_s \prod_{i \text{ odd}}^{2m-1} b_i} \prod_{i \text{ odd}}^{2m-1} h_i^{1/2} \left[\sum_{i \text{ odd}}^{2m-1} h_i^{-1}(dx_i^2 + dx_{i+1}^2) + \cdots + \prod_{i \text{ odd}}^{2m-1} h_i^{-1} dx_5^2 + dx_6^2 + \right. \\
&\qquad \left. + \frac{c_5 N \bar{g}'_s}{u^2} \prod_{i \text{ odd}}^{2m-1} b_i (du^2 + u^2 d\Omega_3^2) \right], \\
h_i &= 1 + a_i^2 u^2 , \\
a_i^2 &= \frac{b_i^2}{c_5 N \prod_{j \text{ odd}}^{2m-1} b_j \bar{g}'_s} , \\
e^{2\phi'} &= \frac{c_5 N}{\prod_{i \text{ odd}}^{2m-1} b_i \bar{g}'_s u^2} \prod_{i \text{ odd}}^{2m-1} h_i^{1/2} ,
\end{aligned} \tag{3.15}
$$

where we have rescaled $x_5 \to \prod_i b_i/(g'_s l'^2_s)^m x_5$ and we have taken into account the fact that under T-duality $\phi \to \phi - \frac{1}{2}\ln(g_{55})$. Note that unlike the type IIB NS5 branes background where $m = 1, 2, 3$ here $m = 1, 2$. The 3-form field A (up to numerical factors) takes the form

$$
\begin{aligned}
m = 1: &\qquad A_{346} \sim \frac{(g'_s l'^2_s)^2}{b^2} a^2 u^2 , &\qquad A_{125} &\sim \frac{(g'_s l'^2_s)^2}{b^2} \frac{a^2 u^2}{1 + a^2 u^2} , \\
m = 2: &\quad A_{i,i+1,6} \sim \frac{(g'_s l'^2_s)^3}{b_i^3} \frac{a_i^2 u^2}{1 + a_i^2 u^2} , &\qquad A_{i,i+1,5} &\sim \frac{(g'_s l'^2_s)^3}{b_i \prod_j b_j} \frac{a_i^2 u^2}{1 + a_i^2 u^2} .
\end{aligned} \tag{3.16}
$$

The curvature of the metric is the same as for the type IIB NS5 branes (3.14). In the IR the effective string coupling is large and we have to lift the solution to eleven dimensions. The background becomes that of wrapped M5 branes. As we increase the energy we can trust the NS5 branes background which provides a deformation of the wrapped M5 branes background.

4. Wilson loops

In this section will use the dual string description in order to compute Wilson loops (surfaces) for the different brane theories.

4.1 Dp-branes

According to the AdS/CFT correspondence, the expectation value of the Wilson loop operator of the gauge theory can be computed in the dual string description by evaluating the partition function of a string whose worldsheet is bounded by the loop [18, 19]. In the supergravity approximation the dominant contribution comes from the minimal two dimensional surface bounded by the loop. The expectation value of the Wilson loop operator is

$$\langle W(C)\rangle \sim e^{-S}\,, \tag{4.1}$$

where S is the string action evaluated on the minimal surface. We will use the same prescription in the case of a nonzero B field. The string action reads now

$$S = \frac{1}{2\pi l_s^2}\int d\tau d\sigma \sqrt{det g_{ij}} + \frac{1}{2\pi l_s^2}\int B_{ij}\partial_\tau X^i \partial_\sigma X^j\,, \tag{4.2}$$

where $g_{ij} = \partial_i X^\mu \partial_j X^\nu G_{\mu\nu}$ is the induced metric.

Consider a static $Q\bar{Q}$ configuration. In general the quark and antiquark move with velocity $\bar{p} = \frac{b}{l_s^2}p$. When $B = 0$, in the $l_s \to 0$ limit, the velocity appears via a multiplicative factor in $Q\bar{Q}$ potential, as expected by the Lorentz symmetry. When $B \neq 0$ the situation is different. There is no Lorentz symmetry and the B field term contributes. When the strings are not moving the end points of strings cannot be fixed at a finite distance L from each other at large u [8] since they grow with u. The endpoints of the strings can be fixed at large u as follows. As was noted in [12], the interaction of charges of opposite sign in a magnetic field is nonlocal in the sense that the interaction point in terms of the center of mass coordinate is shifted by a momentum dependent term. This suggests that we should use a moving coordinates frame in the computation.[5] Indeed, as seen from (2.3), the end points of the open strings attached to the boundary are quark and anti-quark moving with the same velocity (2.3).

In the following we will consider Dp-branes with $b_i = b$, $i = 1, 3, \ldots, 2m-1$. However we will write the final result for arbitrary b_i. We distinguish two cases. In the first case the rank of the B field is not maximal, thus some of the coordinates are commutative and the loop is parametrized by these. In this case the computation proceeds exactly as in the $B = 0$ case. In the second case the loop is parametrized by the noncommutative coordinates. We will discuss this case. We parametrize the

[5]We note that in the case $m = \frac{p+1}{2}$ we will not be able to fix the end points of the strings at infinity. In this case, the time coordinate x_{p+1} is noncommutative coordinate. For a static configuration where the potential is time independent we cannot find an appropriate shift of the time coordinate.

string configuration by $t=\tau$, $u=\sigma$, $x_1=\bar{p}\tau$, $x_2=x(u)$. Equation (4.2) reads now

$$S=\frac{1}{2\pi}\int d\tau du\sqrt{(1-h\bar{p}^2)\left(1+\left(\frac{u}{R}\right)^{7-p}h(\partial_u x)^2\right)}+\frac{1}{2\pi}\int d\tau du\frac{\bar{p}}{b}(au)^{7-p}h\partial_u x\,, \tag{4.3}$$

where R and h are defined in (2.6). It is minimized when

$$\frac{(u/R)^{7-p}h(1-h\bar{p}^2)\partial_u x}{\mathcal{L}_0}+(au)^{7-p}h\frac{\bar{p}}{b}=\text{const.}\,, \tag{4.4}$$

where $\mathcal{L}_0$ is the integrand of the first term in (4.3).

At large u we have

$$\frac{(1/aR)^{7-p}\partial_u x}{\sqrt{1+(\partial_u x)^2/(aR)^{7-p}}}+\frac{\bar{p}}{b}=\text{const.}\,. \tag{4.5}$$

Therefore if we choose the constant in (4.5) to be $\bar{p}/b$ we can fix the position of the string at large u. With this choice equation (4.4) can be solved written as

$$\partial_u x=\frac{\bar{p}}{b}\left(\frac{u}{R}\right)^{-\frac{7-p}{2}}\left(\left(\frac{u}{R}\right)^{7-p}-\left(\frac{\bar{p}}{b}\right)^2\right)^{-1/2}. \tag{4.6}$$

Hence

$$x(u)=\int_{u_0}^{u}\frac{\bar{p}}{b}\left(\frac{u}{R}\right)^{-(7-p)}\left(\left(\frac{u}{R}\right)^{7-p}-\left(\frac{u_0}{R}\right)^{7-p}\right)^{-1/2}, \tag{4.7}$$

where $\partial_u x|_{u_0}\to\infty$[6]

$$(au_0)^{7-p}=\bar{p}^2\,. \tag{4.8}$$

The $Q\bar{Q}$ separation is defined by

$$\begin{aligned}L=x(u\to\infty)&=\int_{u_0}^{\infty}\left(\frac{R}{u_0}\right)^{\frac{7-p}{2}}\left(1-\left(\frac{u_0}{u}\right)^{7-p}\right)^{-1/2}\left(\frac{u_0}{u}\right)^{7-p}\\&=\frac{R^{\frac{7-p}{2}}}{7-p}u_0^{\frac{p-5}{2}}B\left(\frac{1}{2},\frac{6-p}{7-p}\right).\end{aligned} \tag{4.9}$$

Using (4.3) we calculate the energy of the $Q\bar{Q}$ system

$$E=\frac{1}{2\pi}\int_{u_0}^{\infty}\frac{b}{\bar{p}}\partial_u x\left(\frac{u}{R}\right)^{7-p}du\,. \tag{4.10}$$

The integral (4.10) is divergent due to the quark self-energy. It can be regularized as in [18]:

$$\begin{aligned}E&=\frac{1}{2\pi}u_0\frac{1}{7-p}B\left(\frac{1}{2},\frac{-1}{7-p}\right)\\&=-\frac{1}{2\pi}u_0\left(\frac{1}{2}-\frac{1}{7-p}\right)B\left(\frac{1}{2},\frac{6-p}{7-p}\right).\end{aligned} \tag{4.11}$$

[6]In order for u_0 to be N independent we should take the momentum $\bar{p}$ to be N dependent.

Thus,

$$E \sim -\left(\frac{\bar{g}_{YM}^2 N \prod_{i \text{ odd}}^{2m-1} b_i}{L^2}\right)^{\frac{1}{5-p}} . \tag{4.12}$$

When $p < 5$ the potential is attractive. When $p = 5$ L is independent of u_0 and the regularized energy is zero. In the $p = 6$ case we see that the $Q\bar{Q}$ potential is proportional to $-L^2$ which results in a repulsive force. The potential (4.12) is the same as in the $B = 0$ case [20] with $g_{YM}^2 \to \bar{g}_{YM}^2 \prod_{i \text{ odd}}^{2m-1} b_i$. This is presumably expected by the choice of the moving coordinates frame, and also by the map from noncommutative gauge theory to the commutative one [5].

4.2 Non-extremal Dp-branes

In order to compute the expectation value of the Wilson loop operator in the gauge theory at nonzero temperature we will use the non-extremal Dp-branes background. We again take the previous string configuration. We get

$$S = \frac{1}{2\pi}\int d\tau du \sqrt{(1 - hK^{-1}\bar{p}^2)\left(1 + \left(\frac{u}{R}\right)^{7-p} hK(\partial_u x)^2\right)} + \frac{1}{2\pi}\int d\tau du \frac{\bar{p}}{b}(au)^{7-p} h\partial_u x . \tag{4.13}$$

where $K = 1 - (u_T/u)^{7-p}$.

Solving the equation of motion for $x(u)$ and fixing the end points by a constsnt $\bar{p}/b$, we have

$$\partial_u x = \frac{\bar{p}}{b}\left(\frac{u}{R}\right)^{-\frac{7-p}{2}} K^{-1/2}\left(\left(\frac{u}{R}\right)^{7-p} K - \left(\frac{\bar{p}}{b}\right)^2\right)^{-1/2} . \tag{4.14}$$

Thus,

$$x(u) = \int_{u_0}^{u} \frac{\bar{p}}{b} R^{(7-p)} \left(u^{7-p} - {u_0}^{7-p}\right)^{-1/2} \left(u^{7-p} - {u_T}^{7-p}\right)^{-1/2} , \tag{4.15}$$

wher u_0 is the point where the $\partial_u x \to \infty$,

$$(au_0)^{7-p} = (au_T)^{7-p} + \bar{p}^2 . \tag{4.16}$$

Consider two cases:

a) Low momentum: $(au_T)^{7-p} \gg \bar{p}^2$. Here the non-extremality effects are large and we get

$$E \sim -\left(\frac{\bar{g}_{YM}^2 N \prod_{i \text{ odd}}^{2m-1} b_i}{L^2}\right)^{\frac{1}{5-p}} \left[1 + c\left(T\frac{L^2}{\bar{g}_{YM}^2 N \prod_i b_i}\right)^{(7-p)/(5-p)}\right] , \tag{4.17}$$

where c is N independent dimensionless constant. Again, the potential (4.16) is the same as in the $B = 0$ case [20] with $g_{YM}^2 \to \bar{g}_{YM}^2 \prod_{i \text{ odd}}^{2m-1} b_i$.

b) High momentum: $(au_T)^{7-p} \ll \bar{p}^2$. Here the noncommutativity effects are large and we get the noncommutative extremal case result (4.9).

4.3 Wilson surfaces

The computation of the expectation value of a Wilson surface observable amounts in the supergravity approximation to computing the minimal volume of a membrane bounded at infinity by the surface Σ. Consider first the wrapped M5 branes background (3.3).

The case $m = 1$ When the noncommutative effects are large the background (3.3) has three small coordinates x_1, x_2, x_5. There are two cases to distinguish. The first is when the membrane wraps one of this coordinates. In this case the result should coincide with that of the D4 branes Wilson loop computation. The second case is when the membrane is not wrapping one of these small coordinates. This case is similar to the computation of the potential between monopole and antimonopole. Here we expect an end fixing problem since unlike the electric charges in the B field background there is no useful moving coordinate frame.

We start with the first case. We denote the membrane coordinates by τ, σ_1, σ_2. Consider, for instance, the configuration $\tau = x_6, b\bar{R}_0^2\sigma_1 = x_5, \sigma_2 = u, x_2 \equiv x(u)$ and $x_1 = \bar{p}\tau$. σ_1 parametrizes the compactification circle $0 \leq \sigma_1 \leq 2\pi$. The membrane action reads

$$S = \frac{1}{(2\pi)^2}\int d\tau d\sigma_1 du \left\{ \sqrt{(1 - h^{-1}\bar{p}^2)\left(1 + \left(\frac{u^3}{\pi N b\bar{R}_0^2}\right) h^{-1}(\partial_u x)^2\right)} + \frac{\bar{p}}{b}(au)^3 h^{-1}\partial_u x \right\}, \tag{4.18}$$

where here $h = 1 + a^3u^3$. Performing the integration on σ_1 we get (4.3) for $p = 4$, where $R^3 \to \pi N b\bar{R}_0^2$. This is the expected result.

Consider the second case and let the configuration be $\tau = x_6, \sigma_1 = x_3, \sigma_2 = u, x_4 \equiv x(u)$. Since the C_{346} component is nonzero, the C term in the membrane action contributes and we get the action per unit length

$$S = \frac{1}{4\pi^2 b\bar{R}_0^2}\int d\tau du \left\{ \sqrt{h\left(1 + \left(\frac{u^3}{\pi N b\bar{R}_0^2}\right)(\partial_u x)^2\right)} + \frac{a^3}{b}u^3\partial_u x \right\}. \tag{4.19}$$

The equation of motion for $x(u)$ at large u is of the form $\partial_u x \sim$ const., and we have an end fixing problem. As we noted above, a similar end fixing problem arises when considering the a D2 brane ending on D4 branes in order to compute the monopole antimonopole potential when $B \neq 0$.

The case $m = 2$**.** The computation here is similar to the $m = 1$ case when the membrane is wrapping a small coordinate. Taking the configuration $\tau = x_6, b\bar{R}_0^2\sigma_1 = x_5, \sigma_2 = u, x_2 \equiv x(u)$ and $x_1 = \bar{p}\tau$, and integrating the action with respect to σ we get (4.18).

Finally, consider the background (3.8). When the noncommutative effects are large the background (3.8) has three small coordinates x_1, x_2, x_5. Again we distinguish two types of membrane configuration. The first is when the membrane wraps one of this coordinates. A configuration like this is $\tau = x_6, \sigma_1 = x_1, \sigma_2 = u, x_2 \equiv x(u)$ and $x_1 = \bar{p}\tau$. The membrane action per unit length reads

$$S = \frac{1}{(2\pi)^2} \int d\tau du \left\{ \sqrt{u^2(1 - h^{-1}\bar{p}^2)\left(1 + \left(\frac{u^4}{\pi N}\right) h^{-1}(\partial_u x)^2\right)} + \frac{\bar{p}}{b^{3/2}}(au)^6 h^{-1}\partial_u x \right\}, \tag{4.20}$$

where $h = 1 + a^6 u^6$. The equation of motion for $x(u)$ is

$$\frac{u\sqrt{1 - h^{-1}\bar{p}^2}\, h^{-1}\frac{u^4}{\pi N}\partial_u x}{\sqrt{1 + h^{-1}\frac{u^4}{\pi N}(\partial_u x)^2}} + \frac{\bar{p}}{b^{3/2}} a^6 u^6 h^{-1} = \text{const.}. \tag{4.21}$$

By choosing the constant to be $\bar{p}/b^{3/2}$, we can fix the end location of the membrane and we have

$$\partial_u x = \frac{\bar{p}\,\pi N}{b^{3/2}} u^{-2}(u^6 - u_0^6)^{-1/2}, \tag{4.22}$$

where $a^6 u_0^6 = \bar{p}^2$. The distance L which is defined as $x(u \to \infty)$ reads

$$L = \frac{\sqrt{\pi N}}{u_0}\left(\frac{1}{6}\int_0^1 dy\, (1 - y)^{-1/2} y^{-1/3}\right). \tag{4.23}$$

Inserting the solution for $x(u)$ in (4.20) we get the interaction energy per unit length between strings of opposite orientation

$$E = \frac{1}{(2\pi)^2}\int_{u_0}^{\infty} \frac{b^{3/2}}{\bar{p}\pi\, N}\, u^6 \partial_u x\, du \sim -\frac{N}{L^2}. \tag{4.24}$$

This is the same result as for the Wilson surface in the $B = 0$ case [18].

The second case is when the membrane is not wrapping one of these small coordinates. Here we expect an end fixing problem. Indeed consider the configuration $\tau = x_6, \sigma_1 = x_3, \sigma_2 = u, x_4 \equiv x(u)$. The membrane action per unit length reads

$$S = \frac{1}{(2\pi)^2}\int d\tau du \sqrt{u^2 h\left(1 + \left(\frac{u^4}{\pi N}\right)(\partial_u x)^2\right)} + \frac{a^6}{b^{3/2}} u^6 \partial_u x\,. \tag{4.25}$$

Writing the equation of motion for $x(u)$ we see that $\partial_u x$ at large u goes like like u and we have an end fixing problem.

5. Discussion

In this paper we studied the Dp-branes supergravity solutions in the presence of a B field, the decoupling limit and various aspects of the correspondence with the noncommutative worldvolume field theories. We analysed the phase structure of the Dp-branes and its dependence on the rank of the B field, i.e. the dependence on the number of noncommutative coordinates. We provided evidence for a possible existence of decoupled Dp-branes worldvolume theories when $p \geq 6$ in presence of a nonzero B field, but clearly more work is required in order to settle this issue [21]. As pointed out [8] the D6 branes system has a negative specific heat. This is usually taken as a sign of instability. However, it may be that the noncommutative effects at high energy require a modification of our field theory understanding of thermal equilibrium. This requires further studies too. The relevance of this to M(atrix) theory compactification on the tori T^p, $p \geq 6$ [22] in the presence of a nonzero B field deserves a further study.

We discussed M5 branes in the presence of nonzero C field. In the case of M5 branes wrapping a circle we found the same decoupling limit discussed in [5] arising from supergravity. In the UV the good description of this system is in terms of D4 branes background, and we did not find a six dimensional field theory description. Considering M5 branes with six flat nocompact worldvolume coordinates we found another decoupling limit and we discussed this possible deformation of the $(0,2)$ SCFT. We also discussed type IIB and type IIA NS5 branes (wrapping a circle) in the presence of nonzero RR fields.

Finally we computed the expectation value of the Wilson loop (surface) operators using the dual supergravity description. We have seen that, in some cases, in the presence of the nonzero B (C) field there is a way to fix the string (membrane) end point (string) by considering a moving coordinates frame in the computation. The results for both extremal and non-extremal Dp-branes (and for the M5 branes) are the same as in the $B=0$ case with $g^2_{YM} \to \bar{g}^2_{YM} \prod_{i \text{ odd}}^{2m-1} b_i$. This is presumably expected by the map from noncommutative gauge theories to the commutative ones [5].

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